PRN:		Course Code :	CET2301

Second Year **B. Tech** End Semester Examination Winter 2024

Differential Equation and Probability

Day and Date: Mon & 2 DEC 2024 Max Marks: **100** 500938

Time : 10.30AM To 01.30PM Duration: **03** hrs

Instructions:
1) All questions are compulsory.
2) Figures to the right indicate full marks
3) **Use of Non- Programmable calculator is allowed**

		Marks	Bloom's Level	CO
Q.1	**Attempt the following**			
a)	Solve $\frac{d^2y}{dx^2}-9y=e^{-3x}+1+e^{3x}$.	08	L3	CO1
b)	Solve $\frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}-6\frac{dy}{dx}=1+x^2$.	08	L3	CO1
	OR			
b)	Solve $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=\cos(2x)$.	08	L3	CO1
Q.2	**Attempt the following**			
a)	Find the directional derivative of $\phi(x,y,z)=xy^2+yz^3$ at the point $A(2,-1,1)$ in the direction of vector $i+2j+2k$.	08	L2	CO2
b)	Prove that the vector $\overline{F}=(x+2y+az)i+(bx-3y-z)j+(4x+cy+2z)k$ is solenoidal and determine the constants a,b,c if $\overline{F}$ is irrotational.	08	L2	CO2
	OR			
b)	A vector field $\overline{F}$ is given by $\overline{F}=3x^2yi+(x^3-2yz^2)j+(3z^2-2y^2z)k$. Show that it is irrotational and hence find its scalar potential.	08	L2	CO2
Q.3	**Attempt any two from the following**			
a)	Obtain the Fourier expansion of $f(x)=x^2$ in the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$.	08	L3	CO2
b)	Obtain the Fourier expansion of $f(x)=x-x^2$ in the interval $-\pi\le x\le\pi$.	08	L3	CO2
c)	Obtain the half range sine series for $f(x)=\begin{cases}2 & ;0\le x\le 1\\ 2-x & ;1\le x\le 2\end{cases}$. Hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$.	08	L3	CO2

Backlog

Q.4 **Attempt any two from the following**

a) Fit a straight line to the following data — 08 L2 CO3

Year x	1951	1961	1971	1981	1991
Production y (000 tons)	10	12	8	10	15

Also estimate the production in 1987.

b) Fit a curve of the form $y = a + bx + cx^2$ to the following data — 08 L2 CO3

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

c) Find the both lines of regressions (y on x and x on y) for the following data — 08 L2 CO3

x	6	2	10	4	8
y	9	11	5	8	7

Q.5 **Attempt any Two of the following**

a) Two dice are rolled at random. Obtain the probability distribution of the sum of the number on them. — 09 L1 CO4

b) In a sampling the mean number of defective bolts manufactured by a machine in a sample of 20 is 2. Determine the expected number of samples out of such 500 samples to contain at least 2 defective bolts. — 09 L2 CO4

c) Fit a Poisson distribution to the following data — 09 L3 CO4

x	0	1	2	3	4	5	6
f	143	90	42	12	9	3	1

Q.6 **Attempt any Two of the following**

a) Show that $u = y^3 - 3x^2y$ is a harmonic function. Find its harmonic conjugate and the corresponding analytic function. — 09 L3 CO5

b) Evaluate $\int_C \frac{e^z}{(z^2+1)} dz$, where C is the circle $|z| = 2$. — 09 L3 CO5

c) Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$, along the path i) $y = x$; ii) $y = x^2$. — 09 L3 CO5

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Backlog